**Systems prognostics via degradation models**

This primer on prognostics and health monitoring (PHM) is based off Gebraeel’s journal paper entitled “Residual-life distributions from component degradation signals: a Bayesian approach.” This article was published in 2007, so more sophisticated methods must have been developed since then. However, the general procedure followed in the paper to use degradation models to build Residual Life Distributions (RLDs) is enlightening and could help guide the reading of more recent papers on the same topic.

The approach taken by the paper comprises 3 steps. The first 3 steps might be performed by a manufacturer who wants to get an idea of the reliability of a device that it manufactures. The subsequent steps are followed by a customer who bought a device from that manufacturer. The customer wants to use that device’s real-time data together with reliability data computed by the manufacturer to compute the device’s RLD.

1. Collect reliability data about a population of identical devices.
2. Fit a parametric model (*aka* “degradation model”) to the reliability data of each tested device. (The model parameters are stochastic because the fit of the model to each device’s data yields different parameter values.)
3. Compute the probability distributions of the model’s stochastic parameters – these are called “prior distributions” because they embed our prior knowledge of the devices’ behavior.
4. Collect real-time data from an operating device.
5. Use the collected real-time data to update the prior distributions into a “posterior distribution” of the degradation model’s stochastic parameters. (Posterior meaning after our state of knowledge is modified through observing new data.) This is where Bayesian updating comes into play: you update the prior distributions into a posterior distribution using Bayes theorem.
6. Compute the RLD using the posterior distribution.

Note: Bayesian updating is a method among many others. However, it’s popular, not only in PHM but also in other fields.

**Steps 1 and 2 details**

In his paper, Gebraeel applies this method to bearings. To collect reliability data, he ran 50 bearings continuously until failure. The data collected from each device is vibration (voltage) signals from accelerometers sampled every 2 minutes. He then observed that the degradation portion of the signal could be fitted as an exponential function (without measurement noise) – see fig. 1. The model chosen is as follows:

Where and are the stochastic parameters mentioned above and is a Brownian motion process modeling measurement errors. is a constant common to all bearings (he sets it to 0 in his paper so not very important) while and are specific to each bearing.

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Description automatically generated

Figure 1. Degradation signal with exponential trend line (from Gebraeel’s PhD thesis (2003))

**Step 3 details**

For each tested bearing, Gebraeel fitted the above exponential model to the collected vibration data. Thus, for each bearing, he computed a value for and a value for . Since he tested 50 bearings, he ended up with 50 values of and 50 values of . He then estimated the mean and variance of and using the sample mean and sample variance. Finally, by plotting the histogram of and , he formulated the hypothesis that they followed normal distributions. He then used the chi-square goodness-of-fit test to confirm that hypothesis. As a result of this step, he ended up with a prior distribution for each stochastic parameter: is distributed as a lognormal (equivalent to saying that is distributed as a normal), is distributed as a normal.

**Step 4 details**

Steps 1 to 3 would be performed by a device’s manufacturer. It would indicate in the device’s datasheet that the degradation signal can be modeled as a growing exponential with stochastic parameters following the prior distributions determined as a result of step 3. A customer would buy one of those devices and integrate it with sensors to collect real-time data while operating.

**Step 5 details**

The prior distributions calculated in step 3 give the device’s buyer prior knowledge of the device’s behavior. These models now need to be specialized to the operating device using real-time operational data. This is done by updating the prior distributions of the stochastic parameters into a joint posterior distribution. For this, Bayes’ theorem is used:

Where is the stochastic degradation model, is the collected real-time data, is the model’s prior distribution, is the likelihood (because it represents the likelihood to observe the collected data given the current model), is called the marginal probability and is a constant which is typically hard to compute but not always necessary in analyses, and is the posterior distribution of the degradation model. represents the updated degradation model after observing the real-time data.

The above equation is applied whenever a new data point is sampled from the device.

**Step 6 details**

Once the posterior distribution is computed, one can deduce the RLD of the operating device. Let random variable be the time until failure from the last sampled datapoint. Failure time is defined as being the first time the degradation signal reaches some user-defined threshold . If is the time at which the last datapoint was recorded, then the device is deemed to have failed when . Let be the k datapoints obtained so far, then, the cumulative distribution function (CDF) of given is what we look for: .

Gebraeel, in his 2007 paper, approximates this as follows:

Using the posterior distribution calculated in Step 5, the right-hand side of the above equation can be computed in closed form under the paper’s assumptions. (However, Gebraeel notes that the method is more general than that and numerical techniques can be used to compute if no closed form is available.)

One then deduces the probability density function (PDF) of by taking the derivative of the expression of with respect to That’s the RLD we are looking for at time . Of course, the above procedure must be done each time a new data point is recorded, at , , etc.

**Additional comments**

A recent review on prognostics methods by Guo et al. classifies prognostics methods into one of (1) Data-driven methods, (2) Physics-based methods (*aka* model-based), or (3) hybrid methods (combining both physics-based and data-driven methods). Data-driven methods exclusively use data generated from sensors to build system health/degradation models (e.g., the "prior distributions" in Gebraeel et al. (2007)); no knowledge of the physics of failure is needed. Physics-based methods are possible through expert knowledge of the physics of failure of a given system; in this case, a mathematical model is typically provided by domain experts (in lieu of prior distributions as in Bayesian-based data-driven methods).

For example, Guo et al. classify artificial neural networks and Bayesian methods (like the one investigated by Gebraeel et al. (2007)) as data-driven methods. The physics-based methods include Kalman filters (and variants) and particle filters. These seem to be the state of the art of prognostics methods. I'm attaching two more papers for your literature review: one using Kalman filters to predict the end of life of batteries (Daigle et al.), and one using Particle filters to predict the crack growth prediction (Corbetta et al.).

From what I've read it seems like most prognostic methods are done in two steps: (1) update of a model of the operating system using latest sensor data, followed by (2) prediction of the remaining useful life using the updated model. For example, (1) is done via Kalman Filtering or Bayesian updating. Step (2) is typically done through computing a probability density function of the remaining useful life.